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Free vibration analysis of micropipe conveying fluid by wave method

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ABSTRACT

The wave method for free vibration analysis of single-wall nanotube conveying fluid is proposed in this paper. The lateral vibration model of nanotube conveying steady flow is derived based on the Timoshenko beam theory. The control partial differential equations are dealt with Fourier transform and the wave form solutions are obtained. The wave propagation matrix and reflection matrices at different boundary conditions are gained. By using the wave train closure principle, the wave characteristic equation for natural frequency computation of nanotube conveying fluid is established. In the example, the natural frequencies of simply supported pipe conveying fluid and the critical velocity of simply supported nanotube conveying fluid are computed and the results show great agreement with that in existing literature. The first five natural frequencies of simply supported and clamped-pinned nanotube conveying fluid are also obtained by the wave method. The influence of fluid velocity on the natural frequency of nanotube conveying fluid is discussed.

1. Introduction

The carbon nanotube (CNT) has been widely used in many fields because of its excellent mechanical and biological properties. It plays an important role in material reinforcement, hydrogen storage, micromechanical oscillators, sensors, etc. [1]. Recently, the interesting dynamic characteristics of nanotube conveying fluid due to the coupling effect between the wall and the fluid have attracted more and more focus [2–4]. The modeling of nanotube is based on several theories. Ansari et al. [5] studied the vibrational character of single-walled carbon nanotube (SWCNT) using gradient elasticity theories and obtained the small-size effect in terms of different gradient elasticity theories. Zheng Yonggang et al. [6] discussed the size and surface effect on mechanic behavior of nanotube in first gradient elasticity. Considering the inertia and strain gradient effects, Wang [7] investigated the wave propagation of single-walled carbon nanotube conveying fluid based on the strain gradient elasticity theory. Nonlocal elasticity theory is another important theory adopted in nanotube modeling. Many researchers investigated the mechanical behavior of nanotube by applying this theory [8,9]. Using the Timoshenko beam theory, Ke et al. [10] investigated the vibration and instability of double-walled carbon nanotubes (DWNTs) conveying fluid based on the modified couple stress theory.

Presently, the stability of nanotube conveying fluid is a hot topic. Vahid Rashidi et al. [11] developed the fluid structure interaction(FSI) model for the stability and vibration of a CNT conveying fluid. They considered the small-size effect of nanoflow field on the stability and dynamic characteristics of nanotube. Haw-Long et al. [12] studied the stability of single-walled carbon nanotube conveying viscous fluid embedded in an elastic media. The influence of fluid velocity, viscosity, etc. on the fundamental frequency of clamped-clamped SWCNT was discussed. Yoon et al. [13] studied the influence of fluid velocity on the stability of nanotube conveying fluid and obtained the critical velocity for instability. They found that the inner high speed fluid could fully affect resonant frequencies of the larger innermost radius long CNT. Lin Wang et al. did much work on the stability of nanotube conveying fluid. He presented a double-elastic beam model for double-wall carbon nanotube (DWNT) conveying fluid based on the Bernoulli–Euler beam theory [14]. They obtained the explicit expressions of natural frequency for simply supported DWNT conveying fluid. At present, he proposed another beam model for the stability analysis of CNT conveying fluid [15]. The nonlocal nanoscale parameter can be considered in his model. When studying the stability of CNT conveying viscous fluid, he found that the effect of fluid viscosity on stability and natural frequency of CNT could be neglected if the continuum beam model was adopted [16]. This conclusion modified the results obtained by Yoon et al. [13,17].

The stability analysis is indeed an important problem of nanotube conveying fluid. However, natural frequency is also an important parameter of nanotube conveying fluid [18]. In this paper, we present a wave method for natural frequency calculation of single-wall nanotube conveying fluid. Firstly, the transverse vibration equation of nanotube conveying steady flow is derived

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based on the Timoshenko beam model. Then, the wave method is applied to analyze the above equation. In the process, we obtained the wave propagation matrix and reflection matrices at three classical boundary conditions. The wave characteristic equation for natural frequency calculation of nanotube conveying fluid was obtained by using the wave train closure principle. Finally, the natural frequencies of nanotube conveying fluid under different fluid velocity were calculated with different boundary conditions.

2. Transverse vibration model of nanotube conveying fluid

The force sketch of nanotube conveying fluid is shown in Fig. 1. Neglecting the friction between nanotube and inner fluid, with the assumption that the fluid is inviscid and incompressible, then the transverse vibration equation is as follows [19].

$$\frac{\partial Q}{\partial x} = m_f\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)^2 y + m_t\frac{\partial^2 y}{\partial t^2} \tag{1}$$

When the nanotube is modeled as Timoshenko beam and considering the rotary inertia of the nanotube cross section, the equation of motion can be expressed as follows

$$\begin{cases} Q = k_0 A_t G\left(\frac{\partial y}{\partial x} - \varphi\right) \\ M = EI\frac{\partial \varphi}{\partial x} \\ \frac{\partial M}{\partial x} + Q = J_t\frac{\partial^2 \varphi}{\partial t^2} \end{cases} \tag{2}$$

where E denotes the Young's modulus; G is the shear modulus; I is the moment inertia of the nanotube cross section; k_0 is the shear coefficient and for a thin wall tube always $k_0 = \frac{2(1+\mu)}{4+3\mu}$ [4]; μ is the Poisson ratio; A_t is the area of the nanotube cross section; J_t is the mass moment of inertia for nanotube; m_t is the mass of nanotube per unit length; m_f is the mass of fluid per unit length; Q is the shear force; M is the bending moment; φ is the rotation angle of nanotube cross section caused by bending deformation; and V is the fluid velocity.

Combining Eqs. (1) and (2) , the following partial differential equation with respect to y can be obtained.

$$\begin{aligned} &EI(k_0A_tG - m_fV^2)\frac{\partial^4 y}{\partial x^4} + [-EI(m_t+m_f) + J_p(m_fV^2 - k_0A_tG)]\frac{\partial^4 y}{\partial x^2\partial t^2} \\ &\quad - 2EIm_fV\frac{\partial^4 y}{\partial x^3\partial t} + 2J_tm_fV\frac{\partial^4 y}{\partial x\partial t^3} + J_t(m_t+m_f)\frac{\partial^4 y}{\partial t^4} \\ &\quad + k_0A_tG(m_t+m_f)\frac{\partial^2 y}{\partial t^2} + 2k_0A_tGm_fV\frac{\partial^2 y}{\partial x\partial t} + k_0A_tGm_fV^2\frac{\partial^2 y}{\partial x^2} = 0 \end{aligned} \tag{3}$$

It is difficult to solve Eq. (3) directly. According to Doyle [20], the Fourier transform pairs of y are introduced, then the problem will be simplified greatly

$$\begin{aligned} &\hat{y}(x,\omega) = \int_{-\infty}^{+\infty} y(x,t)\exp(-i\omega t)dt \\ &y(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{y}(x,\omega)\exp(i\omega t)d\omega \end{aligned} \tag{4a, b}$$

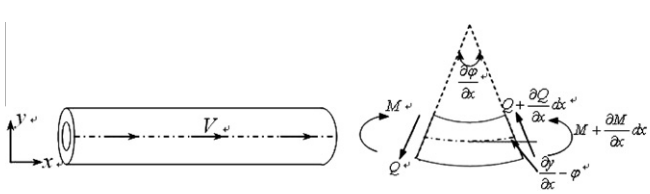

Fig. 1. The sketch of nanotube conveying fluid.

Substituting Eq. (4a,b) into Eq. (3), the following ordinary differential equation will be obtained.

$$\begin{aligned} &EI(k_0A_tG - m_fV^2)\frac{d^4\hat{y}_n}{dx^4} - 2iEI\omega_n m_fV\frac{d^3\hat{y}_n}{dx^3} + \left[EI\omega_n^2(m_t+m_f)\right. \\ &\quad \left. - J_t\omega_n^2(m_fV^2 - k_0A_tG) + k_0A_tGm_fV^2\right]\frac{d^2\hat{y}_n}{dx^2} \\ &\quad + 2i\omega_n m_fV(k_0A_tG - \omega_n^2J_t)\frac{d\hat{y}_n}{dx} + \omega_n^2(m_t+m_f)(\omega_n^2J_t - k_0A_tG)\hat{y}_n = 0 \end{aligned} \tag{5}$$

The solution of Eq. (5) holds the following form.

$$\hat{y} = w\exp(ikx) \tag{6}$$

where, in Eq. (6), w is the undetermined coefficient and k is the wave number.

Substituting Eq. (6) into Eq. (5), the so-called dispersion equation can be obtained.

$$\begin{aligned} &EI(k_0A_tG - m_fV^2)k^4 - 2\omega EIm_fVk^3 - [EI\omega^2(m_t+m_f) \\ &\quad + m_fV^2(k_0A_tG - J_t\omega^2) + J_t\omega^2k_0A_tG]k^2 \\ &\quad - 2\omega m_fV(k_0A_tG - J_p\omega^2)k + \omega^2(m_t+m_f)(J_p\omega^2 - k_0A_tG) = 0 \end{aligned} \tag{7}$$

Obviously, Eq. (7) has four roots. The four values of wave number k denote four wave motions, For convenience purpose, we let subscripts 1 and 2 denote left going wave and the subscripts 3 and 4 denote right going wave in this paper. So Eq. (6) can be rewritten as follows

$$\hat{y} = \sum_{j=1}^{4} w_j\exp(ik_jx) \tag{8}$$

Substituting Eq. (8) into Eq. (2), the parameters φ,Q and M can be obtained in the following form.

$$\begin{cases} \hat{\varphi} = \sum_{j=1}^{4}\lambda_j w_j\exp(ik_jx) \\ \hat{Q} = \sum_{j=1}^{4}\gamma_j w_j\exp(ik_jx) \\ \hat{M} = \sum_{j=1}^{4}\beta_j w_j\exp(ik_jx) \end{cases} \tag{9a—c}$$

where,

$$\begin{aligned} &\lambda_j = ik_j - \frac{i}{k_0A_tG}[(m_t+m_f)\omega^2/k_j + 2m_fV\omega + m_fV^2k_j] \\ &\gamma_j = i(m_t+m_f)\omega^2/k_j + 2im_fV\omega + im_fV^2k_j \\ &\beta_j = EI\left\{-k_j^2 + \frac{1}{k_0A_tG}[(m_t+m_f)\omega^2 + 2m_fVk_j\omega + m_fV^2k_j^2]\right\} \end{aligned}$$

3. Wave propagation and reflection along nanotube conveying fluid

From the view of wave theory, the vibration wave propagates along the nanotube, which reflects and undergoes transition at the discontinuity of material. At the boundary of the nanotube, due to no other medium, the wave is reflected only. As Fig. 2 shows, two points A and B have a distance of x, the wave motions at point A are represented by a^+ and a^-. In the same way, b^+ and b^- denote the wave motion at point B. The relationship between them can be expressed as follows [21].

$$b^+ = T_ra^+, \quad a^- = T_lb^- \tag{10}$$

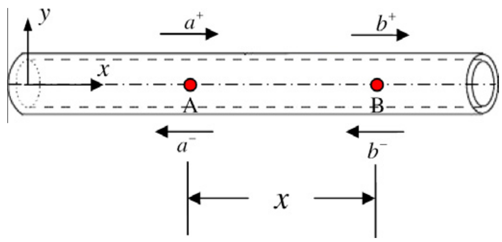

Fig. 2. Wave propagation diagram.

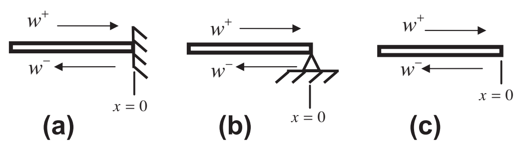

Fig. 3. Wave reflection at boundaries: (a) clamped support, (b) simply support, (c) free end.

Here,

$$a^+ = \begin{Bmatrix} a_3 \\ a_4 \end{Bmatrix}, \quad a^- = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}, \quad b^+ = \begin{Bmatrix} b_3 \\ b_4 \end{Bmatrix}, \quad b^- = \begin{Bmatrix} lb_1 \\ b_2 \end{Bmatrix} \tag{11}$$

T_r and T_l represent the propagation matrix of right and left going waves, it is easy to find from Eqs. (8) and (9a–c) that

$$T_r = \begin{bmatrix} e^{ik_3x} & 0 \\ 0 & e^{ik_4x} \end{bmatrix}, \quad T_l = \begin{bmatrix} e^{-ik_1x} & 0 \\ 0 & e^{-ik_2x} \end{bmatrix} \tag{12}$$

Fig. 3 shows wave reflection at three typical supports, the coordinate of the support is chosen at $x = 0$. Let w^+ and w^- denote the incident wave and outgoing wave and R is the reflection matrix.

The relation between the incident and outgoing waves can be expressed as follows [21].

$$w^- = Rw^+ \tag{13}$$

Here,

$$w^+ = \begin{Bmatrix} w_3 \\ w_4 \end{Bmatrix}, \quad w^- = \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} \tag{14}$$

The reflection matrix R can be obtained in virtue of the force and moment balance condition at the boundary.

Considering the clamped boundary condition,

$$\begin{cases} \hat{y} = 0 \\ \hat{\varphi} = 0 \end{cases} \tag{15}$$

Combining Eqs. (8) and (9a–c) , we can obtain the following equation

$$\begin{cases} w_1 + w_2 + w_3 + w_4 = 0 \\ \lambda_1 w_1 + \lambda_2 w_2 + \lambda_3 w_3 + \lambda_4 w_4 = 0 \end{cases} \tag{16}$$

Introducing Eq. (14) into Eq. (16), then we rewrite Eq. (16) as follows

$$\begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} w^- + \begin{bmatrix} 1 & 1 \\ \lambda_3 & \lambda_4 \end{bmatrix} w^+ = 0 \tag{17}$$

Reminding Eq. (13), we get the reflection matrix at clamped boundary as follows

$$R_c = -\begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ \lambda_3 & \lambda_4 \end{bmatrix} \tag{18}$$

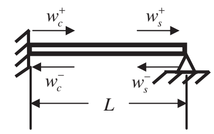

Fig. 4. Wave motion in clamped-pinned nanotube.

In the same way, the reflection matrix R at pinned and free end can readily be obtained.

$$R_s = -\begin{bmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ \beta_3 & \beta_4 \end{bmatrix} \tag{19}$$

$$R_f = -\begin{bmatrix} \beta_1 & \beta_2 \\ \gamma_1 & \gamma_2 \end{bmatrix}^{-1} \begin{bmatrix} \beta_3 & \beta_4 \\ \gamma_3 & \gamma_4 \end{bmatrix} \tag{20}$$

Now, the propagation and reflection matrices have been available, we will use the wave train closure principle to establish the wave equation for natural frequency computation of nanotube conveying fluid.

4. Wave equation of nanotube conveying fluid

Fig. 4 shows a clamped-pinned nanotube conveying fluid. The wave motion at the two ends is represented by $w_c^{\pm}$ and $w_s^{\pm}$. Using the wave train closure principle, the following relation between wave motions can be obtained [21,22].

$$w_s^+ = T_l(L)w_c^+, \quad w_s^- = R_s w_s^+, \quad w_c^- = T_r(L)w_s^-, \quad w_c^+ = R_c w_c^- \tag{21}$$

Eq. (21) can be manipulated into the following form [15,16].

$$(R_c T_l(L) R_s T_r(L) - \mathbf{I}) w_c^+ = 0 \tag{22}$$

Here $T_l(L) = \begin{bmatrix} e^{-ik_1L} & 0 \\ 0 & e^{-ik_2L} \end{bmatrix}$, $T_r(L) = \begin{bmatrix} e^{ik_3L} & 0 \\ 0 & e^{ik_4L} \end{bmatrix}$, $\mathbf{I}$ is the second order identity matrix.

Obviously, if the Eq. (22) has a non-trivial solution, the following condition must be satisfied.

$$h(\omega) = |R_c T_l(L) R_s T_r(L) - \mathbf{I}| = 0 \tag{23}$$

The natural frequency can be obtained from Eq. (23).

5. Examples

5.1. Example 1

Natural frequencies are calculated for simply supported pipe conveying fluid. The model is shown in Fig. 5.

The parameters of pipe and fluid are chosen as follows [23]: Young's modulus $E = 210$ GPa, the outer and inner diameters are $D = 324$ mm, and $d = 292$ mm, respectively. The length of pipe is $L = 32$ m, the density of pipe material is $\rho_t = 8200$ kg/m^3 and the density of fluid is $\rho_f = 908.2$ kg/m^3. The results are listed in Table 1 and compared with that in [23,24].

The first five natural frequencies are computed under three different fluid velocities, that is, $V = 0$, $V = 15$ m/s, $V = 25$ m/s.

It is shown in Table 1 that the relative errors between the results in this paper and that in Refs. [23,24] are less than 0.28%.

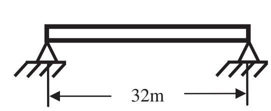

Fig. 5. Simply supported nanotube conveying fluid.

Table 1
Results of the first five natural frequencies of simply supported pipe conveying fluid (rad/s).

Fluid velocity	Natural frequency				
	ω_1	ω_2	ω_3	ω_4	ω_5
V = 0					
Present result	4.3718	17.4705	39.2461	69.6165	108.4682
Xu et al. [23]	4.3732	17.4928	39.3587	–	–
Relative error (%)	0.03	0.12	0.28	–	–
Housner [24]	4.3732	17.4928	–	–	–
Relative error (%)	0.03	0.12	–	–	–
V = 15 m/s					
Present result	4.2857	17.3946	39.1729	69.5445	108.3968
Xu et al. [23]	4.2870	17.4171	39.2858	–	–
Relative error (%)	0.03	0.12	0.28	–	–
Housner [24]	4.2971	17.3922	–	–	–
Relative error (%)	0.26	0.01	–	–	–
V = 25 m/s					
Present result	4.1291	17.2593	39.0426	69.4163	108.2697
Xu et al. [23]	4.1293	17.2816	39.1559	–	–
Relative error (%)	0.004	0.12	0.28	–	–
Housner [24]	4.1576	17.2122	–	–	–
Relative error (%)	0.68	0.27	–	–	–

Table 2
The first six natural frequencies of pinned-pinned nanotube conveying fluid ($\times 10^8$rad/s).

Fluid velocity	Natural frequency					
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
V = 0	0.2825	1.1075	2.4144	4.1204	6.1382	8.3879
V = 20 m/s	0.2497	1.0814	2.3884	4.0929	6.1081	8.3545
V = 40 m/s	0.1219	1.0005	2.3097	4.0098	6.0175	8.2542
V = 45.26	0	9.6912	2.2799	3.9786	5.9835	8.2166

Table 3
The first five natural frequencies of clamped-pinned nanotube conveying fluid ($\times 10^9$rad/s).

Fluid velocity	Natural frequency				
	ω_1	ω_2	ω_3	ω_4	ω_5
V = 0	0.6971	1.9235	3.7331	6.0924	8.9611
V = 25 m/s	0.6968	1.9233	3.7329	6.0922	8.9610
V = 35 m/s	0.9666	1.9231	3.7327	6.0921	8.9608
V = 50 m/s	0.6961	1.9226	3.7322	6.0916	8.9604

So the proposed method is reasonable. The main reason of the above differences is the different beam models adopted between Refs. [23,24] and present paper. In Refs. [23,24], Euler–Bernoulli beam was used, however, the Timoshenko beam is adopted in this paper.

It is clear that the natural frequencies tend to decrease as the fluid velocity increase. The reason is that the increasing fluid velocity can weaken the pipe stiffness.

5.2. Example 2

Natural frequencies of a pinned-pinned nanotube conveying fluid, as shown in Fig. 5, are calculated here. The parameters of nanotube and fluid are as follows [25]: $E = 1.44$ GPa, $\rho_f = \rho_t = 1000\ \mathrm{kg/m^3}$, $\mu = 0.38$, $D = 200\ \mu\mathrm{m}$, $d = 160\ \mu\mathrm{m}$, $L = 4000\ \mu\mathrm{m}$. We calculate the first six natural frequencies under four different fluid velocities, that is, $V = 0$, $V = 20$ m/s, $V = 40$ m/s, $V = 45.26$ m/s.

The first six natural frequencies of nanotube conveying fluid are listed in Table 2. The curves of real and imaginary part of $h(w)$ versus frequency are shown in Fig. 6. When the real and imaginary

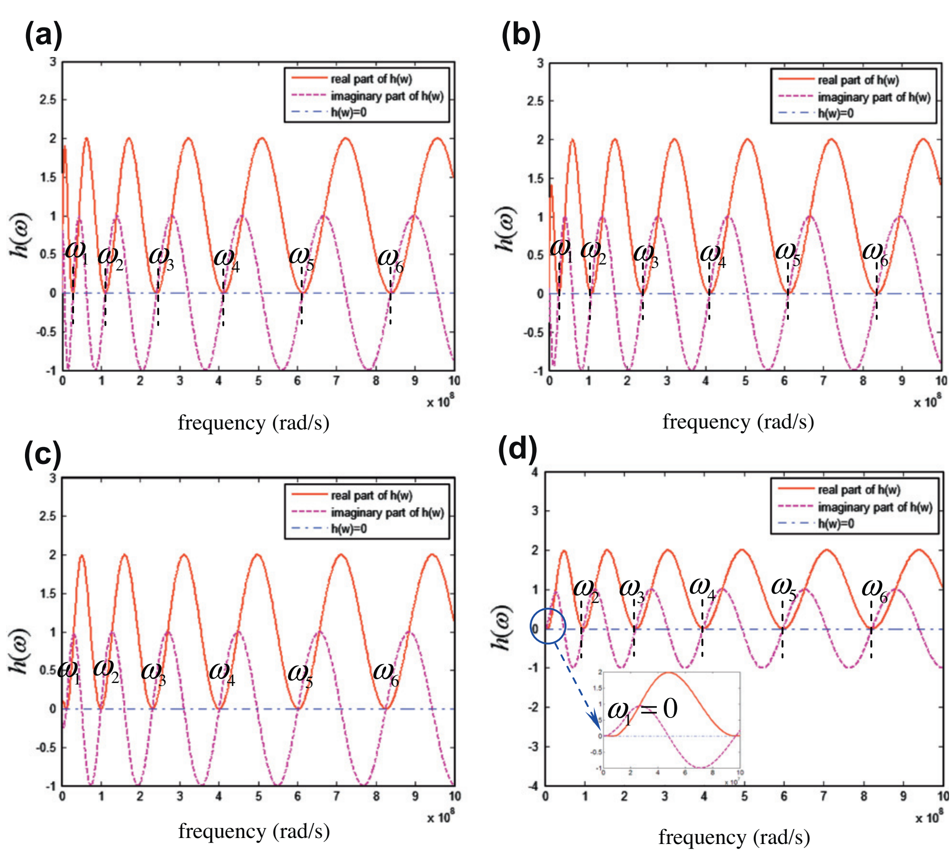

Fig. 6. The first six natural frequencies of pinned-pinned nanotube conveying under different fluid velocities: (a) $V = 0$; (b) $V = 20$ m/s, (c) $V = 40$ m/s, (d) $V = 45.26$ m/s.

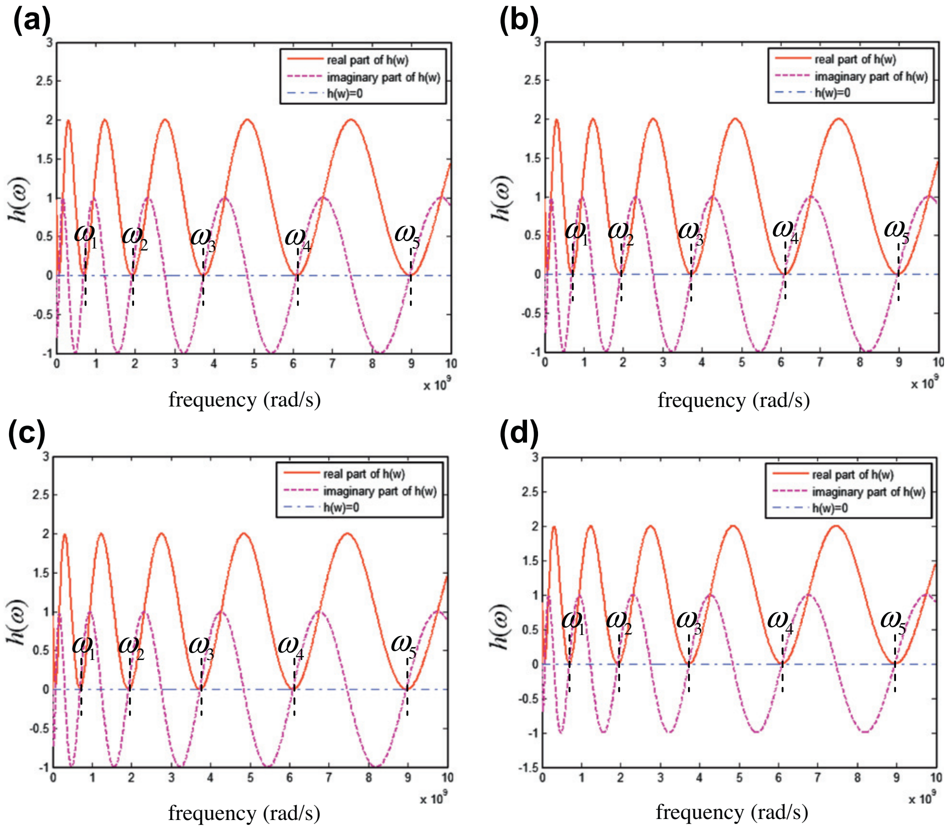

Fig. 7. The first five natural frequencies of clamped-pined nanotube conveying fluid under different fluid velocities: (a) $V = 0$, (b) $V = 25$ m/s, (c) $V = 35$ m/s, (d) $V = 50$ m/s.

part of $h(w)$ both are equal to zero, the corresponding frequency point is the natural frequency point.

From Table 2 and Fig. 6, we can find that the natural frequency decreases with increasing fluid velocity. When the fluid velocity reaches 45.26 m/s, the first natural frequency vanishes, then divergence instability occurs. The corresponding fluid velocity is named critical velocity. The critical velocity of the above nanotube conveying fluid is 45.262 m/s [25]. We can find that a great agreement is shown between the results of critical velocities. It also validate the proposed wave method for the natural frequency of nanotube conveying fluid.

5.3. Example 3

Another example for natural frequency calculation of clamped-pinned nanotube conveying fluid (shown in Fig. 4) is computed. The parameters of nanotube and fluid are as follows: $E = 1$ TPa, $\rho_f = 1000$ kg/m^3, $\rho_t = 2240$ kg/m^3, $\mu = 0.28$, $D = 100$ μm, $d = 80$ μm, $L = 4000$ μm.

The results of the first five natural frequencies are listed in Table 3. The natural frequency points are shown in Fig. 7.

We can also find that as the fluid velocity increases, the natural frequencies of nanotube conveying fluid decrease correspondingly. The reason is that the effect stiffness of nanotube is lost as the fluid velocity increases.

6. Concluding remarks

The wave method for natural frequency computation of nanotube conveying fluid is proposed in this paper. The Timoshenko beam model is applied to obtain the lateral vibration of nanotube conveying fluid. The wave propagation and reflection model is established at three different typical boundary conditions. The wave equation to calculate natural frequency of nanotube conveying fluid is derived based on the wave train closure principle. Three examples are presented for natural frequency calculation and the comparisons are conducted. Both the results of natural frequency and critical velocity have shown great agreement with that in published references. The results have shown that the natural frequencies of nanotube conveying fluid decrease with the fluid velocity increasing. Once the first natural frequency decreases to zero, divergence instability occurs.

Acknowledgment

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